$x=\frac{5}{4}$. This value of x does not satisfy B, neither does it satisfy the original equation.

By studying the two operations of squaring, in the solution of B, it will be seen that extraneous equations were introduced. The squaring the first time was equivalent to multiplying B by $\sqrt{(x-1)}+\sqrt{(x+1)}+1$, that is, the equation resulting from squaring the first time is equivalent to

$$[\sqrt{(x-1)}+\sqrt{(x+1)}+1][\sqrt{(x-1)}+\sqrt{(x+1)}-1]=0\ldots.(1),$$

and the second operation of squaring which gives the equation $1=4(x-1)$ is equivalent to multiplying (1) by $[\sqrt{(x-1)}-\sqrt{(x+1)}+1][\sqrt{(x-1)}-\sqrt{(x+1)}-1]$, that is, the equation $1=4(x-1)$ is equivalent to the equation

$$[\sqrt{(x-1)}+\sqrt{(x+1)}+1][\sqrt{(x-1)}+\sqrt{(x+1)}-1]$$
$$[\sqrt{(x-1)}-\sqrt{(x+1)}+1][\sqrt{(x-1)}-\sqrt{(x+1)}-1]=0.$$

This equation, therefore, is equivalent to the system of equations

$$\left\{\begin{array}{l}\sqrt{(x-1)}+\sqrt{(x+1)}+1=0=P\\ \sqrt{(x-1)}+\sqrt{(x+1)}-1=0=Q\\ \sqrt{(x-1)}-\sqrt{(x+1)}+1=0=R\\ \sqrt{(x-1)}-\sqrt{(x+1)}-1=0=S\end{array}\right\}$$

Of these, the only one that is satisfied by the value $x=\frac{5}{4}$ is R. The solution, therefore, of any one of these equations gives the same value of x, viz., $x=\frac{5}{4}$, and this value of x satisfies R only.

This whole subject of *derivation of equations* has been neglected by most writers on algebra in America until quite recently. The recent texts of Fisher and Schwatt, Beman and Smith, and several others have given considerable attention to the subject.

Professor Chrystal has given a very good treatment of the subject in his Algebra, Vol. I, § XIV. On page 285, he says, "There are few parts of algebra more important than the logic of the derivation of equations, and few, unhappily, that are treated in a more slovenly fashion in elementary teaching."

This problem was also solved in a very excellent manner by *H. C. WHITAKER, G. B. M. ZERR, COOPER D. SCHMITT. W. H. CARTER, W. W. LANDIS, CHARLES C. CROSS, J. M. BOORMAN, J. D. CRAIG, A. F. KOVARIK, ELMER SCHUYLER*, and *J. SCHEFFER*.

110. **Proposed by J. C. CORBIN, Pine Bluff, Ark.**

Put down any number of pounds, shillings and pence under £11, taking care that the number of pence is less than the number of pounds. Reverse this sum, putting pounds in the place of pence, and subtract from the original. Again reverse this remainder and add. The result in all cases will be £12 18s 11d, neither more nor less, whatever the amount with which we start. Verify and explain.

I. Solution by Prof. N. F. DAVIS, Brown University, Providence, R. I.; J. D. CRAIG, New Germantown, N. J.; the late SYLVESTER ROBBINS, North Branch Depot, N. J.; J. SCHEFFER, A. M., Hagerstown, Md.; COOPER D. SCHMITT, A. M., University of Tennessee, Knoxville, Tenn.; and H. C. WHITAKER, Ph. D., Manual Training School, Philadelphia, Pa.

	£	s.	d.
1st	a	b	c
Or thus	$a-1$	$19+b$	$12+c$
Reverse	c	b	a
Subtract	$a-c-1$	19	$12+c-a$
Reverse	$12+c-a$	19	$a-c-1$
Add	12	18	11

II. Solution by J. W. YOUNG, Oliver Graduate Scholar in Mathematics, Cornell University, Ithaca, N. Y.; WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; and G. B. M. ZERR, A.M., Ph.D., The Temple College, Philadelphia, Pa.

The restrictions placed on the problem are unnecessary. The proposition is true for any sum of pounds, shillings and pence, provided, when we subtract we take care to take the positive difference of the two sums obtained as described. The proposition may be proved thus:

Let a, b, c be a number of pounds, shillings and pence, respectively, and suppose $a>c$.

	£	s.	d.	
The two sums are	a	b	c	....(1).
	c	b	a	....(2).

Since $a>c$, before subtracting we must write (1) in the form

$$a-1 \qquad 19+b \qquad 12+c$$

Subtract (2), and we have

	$a-c-1$	19	$12+c-a$
Reverse	$12+c-a$	19	$a-c-1$
Add	£12	18s.	11d., the required result.

III. Solution by BENJAMIN F. YANNEY, A. M., Mount Union College, Alliance, O., and W. W. LANDIS, A. M., Dickinson College, Carlisle, Pa.

Let x stand for the number of pounds, y the number of shillings, and z the number of pence Also, let $x>z$ and $<z+12$, Of course, $y<20$, and $z<12$.

Then performing the operations as indicated, we have,

1.	£x	ys.	zd.
	£z	ys.	xd.
	£$(x-1-z)$	19s.	$(z+12-x)$d.
2.	£$(x-1-z)$	19s.	$(z+12-x)$d.
	£$(z+12-x)$	19s.	$(x-1-z)$d.
	£12	18s.	11d.

Note.—In the case of the decimal system of notation, under similar conditions we find the result to be always 1089. In general, in any system of notation in which n units of the first order make one of the second order, and m units of the second order make one of the third order; if, also, x stands for the number in the third order, y the second order, and z the first order, where $x>z$ and $<z+n$, then proceeding as indicated in the problem, we shall always find the result to be n units of the third order, $n-2$ units of the second order, and $n-1$ units of the first order.

IV. **Solution by E. L. SHERWOOD, A. M., Professor of Mathematics, Beaver College, Beaver, Pa.**

This problem is an example of a class wherein the given quantities are caused to disappear by subtraction or cancellation—the resulting quantity depending on special circumstances, in this case the table ratios.

That the result will always be 12£ 18s 11d may be shown as follows:

	£	s.	d.
Given sum	a	b	c
Reversed	c	b	a
Remainder	$(a-1)-c$	19	$12+c-a$
Remainder reversed	$12+e-a$	19	$a-c-1$
Result	12	18	11

The sum need not be "under 11£" as was given in the problem, but up to a maximum of 23£ 19s 11d. More cannot be taken, as in the first subtraction we would have to "borrow" 2s instead of 1, or, in other words, use 24 as a ratio instead of 12.

If, however, we rigidly adhere to 20 and 12 as ratios, and use them, sums could be taken at random. As

£	s.	d.
100	30	25
25	30	100
74	19	−63
−63	19	74
12	18	11

A similar problem could be proposed with any table—the result varying with the constants of the table.

In general—if r and r' be the ratios (as 12 and 20) the result will be r of the highest, $r'-2$ of the next, and $r-1$ of the lowest denomination.